Internal Wave Action Model

Peter Muller
University of Hawaii
Department of Oceanography
1000 Pope Road, MSB 429
Honolulu, HI 96822
phone: (808)956-8081 fax: (808)956-9164 email: pmuller@hawaii.edu

Grant Number: N00014-96-1-0489

LONG-TERM GOAL

The long term goal of the research project is the understanding of the oceanic internal gravity wave field as a balance of generation, transfer and dissipation processes, and the construction of a numerical dynamical model that will predict changes of the internal wave field in response to changes in the forcing and environmental fields. The internal wave model will be based on the integration of the radiation balance equation.

OBJECTIVES

The objectives of the current project are to:

- Derive a physically motivated form of the sink term in the radiation balance equation that describes the dissipation of wave energy by wave breaking, and
- Assess whether the free parameters of this form can be determined by reasonably well-posed inverse problems.

APPROACH

The approach is mainly theoretical and numerical. The radiation balance equation describes changes of the action density spectrum of the internal wave field along wave group trajectories caused by generation, transfer, and dissipation processes. The predicted quantity is the action density spectrum as a function of wavenumber, position, and time. Because dissipation by wave breaking is a highly nonlinear process its sink term in the radiation balance equation cannot be derived fully from the basic hydrodynamic equations. Instead, such a derivation only arrives at a functional form that contains free parameters that need to be calibrated, by comparison with observations or the results of Large Eddy Simulation (LES) models.

For the reflection off a straight slope it can reasonably be assumed that dissipation and propagation are the main processes. Solutions of the reduced radiation balance equation can then be constructed and their parameter dependence studied.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **30 SEP 2003**	2. REPORT TYPE	3. DATES COVERED **00-00-2003 to 00-00-2003**

4. TITLE AND SUBTITLE **Internal Wave Action Model**	5a. CONTRACT NUMBER
	5b. GRANT NUMBER
	5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER
	5e. TASK NUMBER
	5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **University of Hawaii,,Department of Oceanography,1000 Pope Road, MSB 429,,Honolulu,,HI, 96822**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S)
	11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT
The long term goal of the research project is the understanding of the oceanic internal gravity wave field as a balance of generation, transfer and dissipation processes, and the construction of a numerical dynamical model that will predict changes of the internal wave field in response to changes in the forcing and environmental fields. The internal wave model will be based on the integration of the radiation balance equation.

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**7**	

Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18

WORK COMPLETED

1. Functional form of the dissipation sink term. Based on theoretical arguments by Hasselmann (1974) and Komen et al. (1994) a dissipation sink term of the form

$$S_{diss} = - c(\boldsymbol{k})\, f\,(Ri^{-1})\, E(\boldsymbol{k})$$

has been derived. Here $\boldsymbol{k}$ is the wavenumber vector, $E(\boldsymbol{k})$ the energy density spectrum, Ri the Richardson number, and c and f are two functions. The function $c(\boldsymbol{k})$ determines which wavenumbers get dissipated by breaking events. We have chosen the functional form

$$c(\boldsymbol{k}) = c_0 \begin{cases} 1 & \text{if } m < m_* \\ (m / m_*)^q & \text{if } m \geq m_* \end{cases}$$

with three adjustable parameters c_0, m_* and q, and vertical wavenumber m. This formula reproduces the formula of Garrett and Gilbert (1988) when q approaches infinity. The function $f\,(Ri^{-1})$ determines the overall intensity of wave breaking and dissipation. We assume f to be a monotonic function of Ri^{-1} and have chosen the form

$$f\,(Ri^{-1}) = (Ri^{-1})^p$$

with an adjustable parameter p. The smaller the Richardson number the more vigorous is wave breaking.

2. Solution of radiation balance equation for the reflection off a straight slope. To determine the free parameters c_0, m_*, and q, p the reflection of an incoming Garrett and Munk internal wave spectrum off a straight slope has been analyzed. Because of critical reflection the reflected spectrum has (infinitely) high shear and zero Richardson number with associated vigorous wave breaking. The dominant balance in the radiation balance equation can therefore assumed to be between propagation and dissipation

$$v_n \partial\, E(k) / \partial x_n = S_{diss}$$

where $\boldsymbol{v}$ is the group velocity and $\boldsymbol{x}$ the position. The subscript n denotes the component normal to the slope. This reduced radiation balance equation has been solved for different values of the parameters c_0, m_*, q and p.

The investigation of the four-dimensional parameter space is facilitated by a certain separability of the problem. First, one can introduce a scaled distance

$$\xi_n(x_n) = c_o \int_o^{x_n} dx_n' \left[Ri^{-1}(x_n')\right]$$

which is determined by c_0 and p. The relation between the energy $E(x_n)$ (or any other quantity) and the inverse Richardson number $Ri^{-1}(x_n)$ thus depends only on m_* and q. An example is Figure 1, which shows $E(Ri^{-1})$ for q=0 and q = 2 and $\mu = m_*\, b$ =50 (b=1.3km).

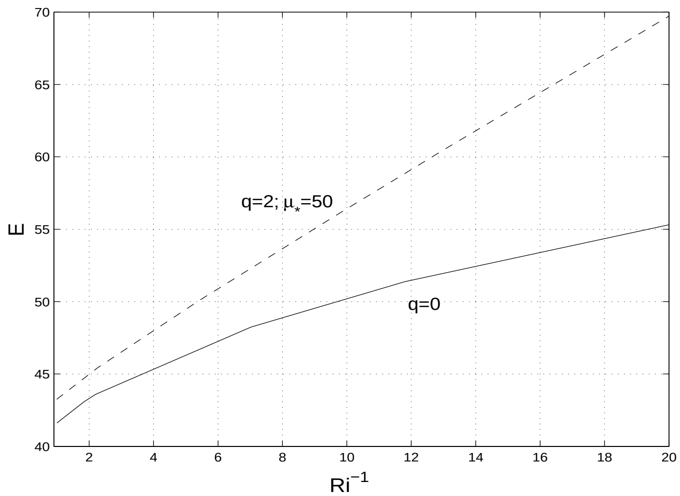

Figure 1: Energy density E vs. Ri^{-1} for q=0 and q =2 and $\mu_* = 50$. Environmental parameters are f / N=0.06,
slope angle γ=0.1, N =5.24 ·10^{-5} s^{-1}. The overall incident flux is 17.3 mW m^{-2}.

The dependence of Ri^{-1}(or any other quantity) on x_n is determined by the parameters c_0 and p, where c_0 determines the distance $x_{n,crit}$ where the inverse Richardson number decays to its critical value and p determines the shape of this decay. Fig. 2 shows Ri^{-1} ($x_n / x_{n,crit}$) for various values of p.

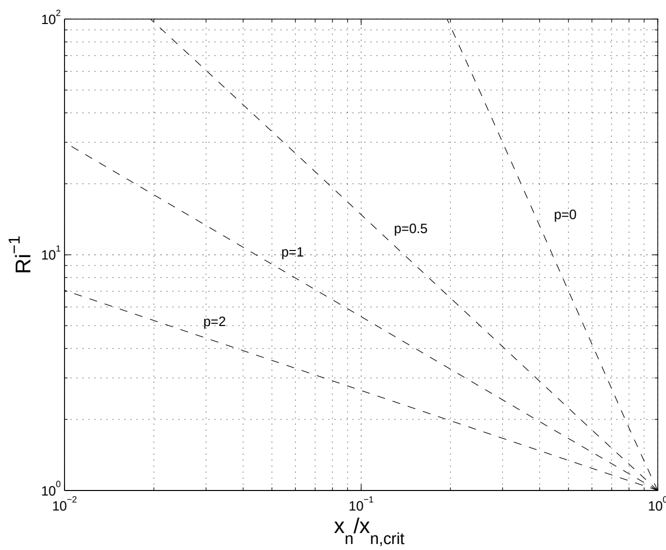

Figure 2. The inverse Richardson number Ri^{-1} as a function of $x_n / x_{n,crit}$ for various values of p. Environmental parameters are f / N =0.06, γ =0.1, N=5.24 ·10^{-5} s^{-1}. The overall incident flux is 17.3 mW m^{-2}.

RESULTS

The form of the dissipation function in the radiation balance equation needs to be specified before any meaningful numerical experiments can be attempted. We have put forward a theoretically motivated form of this dissipation function with four free parameters. We considered the reflection problem off a

straight slope where it might reasonable be assumed that dissipation is balanced by propagation. Analysis of the solutions as a function of the four parameters gave the following results:

1. The total dissipation does not depend sensitively on the choice of the parameter values and is consistent with the physical argument that all the energy flux reflected past the “critical wavenumber” is dissipated. Figure 3 shows, that the total dissipation varies indeed by only twenty percent when m_* is changed by more that an order of magnitude.

2. The parameters determine, however, where in physical and wavenumber space the dissipation occurs and how the resulting spectra look like. Figure 4 shows the vertical wave number spectrum of the inverse Richardson number at $x_{n,crit}$ for two different values of q.

3. The solutions are sufficiently sensitive to the values of the parameters that meaningful inverse problems are obtained when the theoretical solutions are compared to observations.

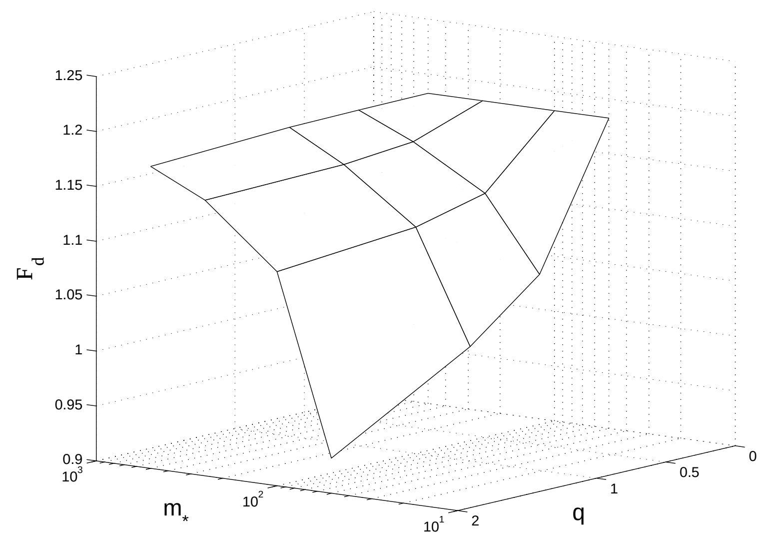

Figure 3. Total dissipation F_d as a function of m_* and q. Environmental parameters are f / N=0.06, slope angle γ=0.1, N =5.24 ·10^{-5} s^{-1}. The overall incident flux is 17.3 m W m^{-2}.

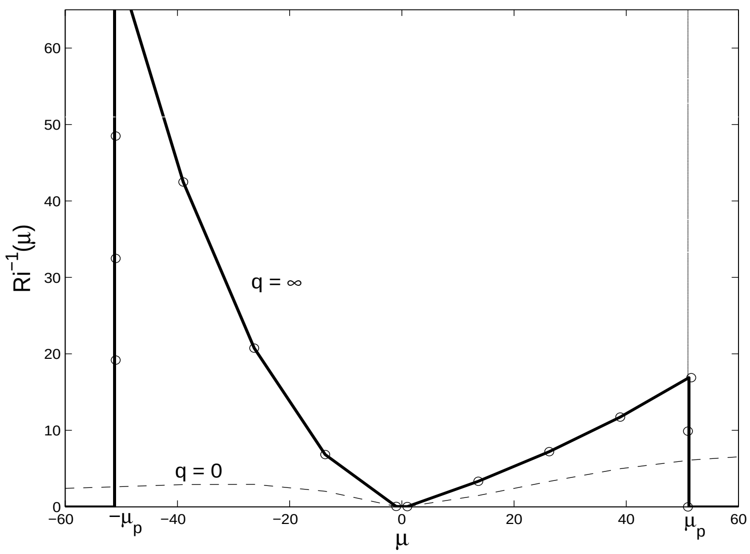

Figure 4. Vertical wavenumber spectrum of inverse Richardson number Ri^{-1} at $x_{n,crit}$ for $q = \infty$ (thick line) and $q = 0$ (dashed line). μ_p is the vertical cut-off wavenumber.

IMPACT/APPLICATION

The development of a predictive dynamical model of internal wave fields will have many benefits and applications.

Internal wave research will benefit from such a model since

- it will provide understanding of the internal wave field as a balance of generation, transfer and dissipation processes,
- it will focus research (it is expected that the proposed model will do for internal wave dynamics what the GM model did for internal wave kinematics), and
- it will predict changes of the internal wave field in response to changes in the forcing and environmental fields.

The dynamical internal wave model can be run in conjunction with circulation models, turbulence models, chemical tracer models, and biological population models where it would predict the internal wave induced transports, dispersion and mixing. In conjunction with acoustic transmission models the model would predict the internal wave induced "noise."

REFERENCES

Garrett, C. and D. Gilbert. 1988. Estimates of vertical mixing by internal waves reflected off a sloping bottom. In: "Small Scale Turbulence and Mixing in the Ocean". Proc. 19th Liège Colloq. on Ocean Hydrodynamics, 405-424.

Hasselmann, K., 1974: On the spectral dissipation of ocean waves due to white capping. *Boundary Layer Met.*, **6**, 107-127.

Komen, G.J., L. Cavaleri, M. Donelan, K. Hasselmann, S. Hasselmann, and P.A.E.M. Janssen, 1994: Dynamics and Modelling of Ocean Waves. Cambridge University Press.

PUBLICATIONS

Guiles, M. The Impact of the Horizontal Coriolis Component on Waves in the Ocean. Masters Thesis, Department of Oceanography, University of Hawaii, May 2003.

Müller, P. and Natarov, A. 2003: The internal wave action model (IWAM). Proceedings, 'Aha Huliko'a Hawaiian Winter Workshop, School of Ocean and Earth Science and Technology, Special Publication, (to be published).

Müller, P., J. McWilliams and J. Molemaker, 2002: Routes to dissipation. The 2D/3D turbulence conundrum. In: *Marine Turbulence.* Springer Verlag (accepted)

Natarov, A. and Müller, P. 2003: A dissipation function for IWAM. (in preparation)

Wang, D and P. Müller, 2002: Equatorial Turbulence. In: *Marine Turbulence*. Springer Verlag (accepted)

Wang, D. and P. Müller, 2002: Effects of equatorial undercurrent shear on upper ocean mixing and internal waves. *J. Phys. Oceanogr.* **32**, 1041-1057.